**Small-Basket Minimum Expected Variance Portfolios of DJIA Stocks: A Case for Active Management   
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The focus of this research is on the performance of portfolios constructed from stocks that make up the Dow Jones Industrial Average (DJIA) using a long-only (no short sales allowed) small-basket minimum expected variance portfolio **(SBMVP) strategy**. The results suggest investors may be able realize enhanced performance relative to the 30-stock DJIA by constructing minimum expected variance (low expected volatility) portfolios of fewer than 10 DJIA stocks.

**Why focus on Small-Basket Portfolios?**

**1.** While it is true in the context of modern portfolio theory that limiting the pool of available investments cannot result in an *ex post* (based on realized returns) efficient frontier that provides a lower risk/return ratio than the market portfolio, actual investment portfolios are formed on *an ex ante* (based on expected returns) basis.

**2.** There are individual investors and portfolio managers who take an *active* approach to investing by constructing small-basket portfolios.

**- J. Dowe Bynum, a co-portfolio manager of the Birmingham, Alabama-based Cook and Bynum Fund (ticker: COBYX), and his partner Richard Cook,** often hold **fewer than 10** stocks at any given time and are willing to stake 20 percent or more of the portfolio on a single stock.

**- Matthew Reiner of Wela Strategies** was quoted in Barron’s as saying that in analyzing a fund for investment he **wants to see the top 10 or 15 holdings (in any fund)** - everything else he calls contamination in terms of what drives performance.

**3.**  Some of **your clients** may want to hold a small number of individual stocks in their investment portfolios in addition to mutual funds and ETFs.

**Why Focus on Low Volatility?**

**1.** The historical performance of different asset classes over the long term is indeed consistent with the theory that higher risk is associated with higher returns. However, it appears that this principle does not always extend to actively managed portfolios. Studies by Clarke, de Silva and Thorley (2006), Blitz and Vliet (2007) and Banner, Papathanakos, and Whitman (2012) **indicate that actively managed low-volatility portfolios can outperform high-volatility portfolios. Because this conflicts with traditional financial theory, it is usually referred to as the *“low-volatility anomaly.”***

- The number of low expected volatility (minimum expected variance) equity products are growing and generally fall into one of two categories:

**Category I portfolios are composed of stocks that individually demonstrate historical low-volatility**

**Category II portfolios are composed of stocks that are expected to display low volatility and/or enhanced risk/return performance based on their expected return and pairwise covariance properties (constructed using mean variance optimization – MVO)**

- The **S&P 500 Low Volatility Index** is an example of a benchmark that tracks the hypothetical value of a portfolio using the **Category I strategy**. On the retail side, **Invesco PowerShares offers a low-volatility ETF (SPLV)** designed to replicate the S&P 500 Low Volatility Index.

- The **MSCI’s Minimum Volatility Indexes are examples of portfolios is Category II that use the MVO methodology. IShares** is bringing to NYSE Arca three exchange-traded funds catering to investors who want to mimic the **MSCI MVO methodology**. The **iShares MSCI Japan Minimum Volatility ETF**, the **iShares MSCI Europe Minimum Volatility ETF** and the **iShares MSCI Asia ex-Japan Minimum Volatility ETF** seek to buy equity securities with characteristics that give them less volatility compared with the broader market.

**2.** A low-volatility strategy may be appropriate for some of **your clients**.

**Method for Constructing SBMVPs**

**1. The SBMVP (small-basket low volatility) strategy** involves using constrained mean variance optimization to form portfolios from those DJIA stocks that offer the potential for minimum expected volatility as does the MSCI approach that was mentioned earlier.

**-** The DJIA is chosen as the larger index portfolio in this study in order to demonstrate the SBMVP strategy relative to a well-known equity index.

- The SBMVP strategy could also be applied to any other group of stocks that exist in the market.

***“Larsen, Glen A., Enhancing the Returns of SRI Portfolios Using a Minimum Variance Small-Basket Strategy,” Journal of Financial Planning, Vol. 26, No., May 2013, 46–53.***

***“An Optimization Strategy for Enhancing the Performance of Fund-of-Funds Portfolios,” with B. Resnick, The Journal of Portfolio Management, Vol. 38, No. 2, Winter 2012, 147-154.***

**2.** Monthly total return data files for the individual stocks in the DJIA are obtained from The University of **Chicago Center for Research in Security Prices (CRSP)**. The CRSP total monthly return data files are then used to calculate the compounded annual rate of return for each of the stocks in the DJIA. Monthly t**otal returns can also be calculated** for individual stocks by downloading the **Monthly Adjusted Closing Price data** into an Excel spreadsheet that can be found in **Yahoo Finance**. The Yahoo monthly price data can then be used calculate total annual returns.

**3.** The **Solver tool in Excel is used to implement MVO** over the previous eight years of return data to solve for the set of individual stock weights used to construct the SBMVPs for the next one-year holding period. For example, the ex ante portfolio for 1995 is constructed based on the stock weights from the ex post efficient portfolio derived using the historical return, standard deviation, and correlation data for the stocks in the DJIA over the previous eight years from 1987–1994.

**4.** Because individual stock weights in the SBMVPs are constrained to 20 percent, the number of stocks in each SBMVP is generally limited to no less than five stocks and no more than 10 stocks. As such, the SBMVP strategy does not depend on owning all (or a large number) of the stocks in the DJIA.

**Results**

**1.** **Table 1** shows that in terms of average returns and standard deviation risk, the **SBMVP strategy generates a higher average annual return and a lower standard deviation of annual returns than the DJIA over the study period.** The results are based on rebalancing only once a year and holding fewer than 10 stocks, which does not take a great deal of work for the CFP advisor.

**Table 1.  
Long Only Small-Basket Minimum Expect Volatility Portfolio (SBMVP) Performance Results 1995 - 2014**

**Traditional Measures of Return and Risk:**

**SBMVP DJIA**  
Average Annual Return 14.32% 11.85%Standard Deviation 15.68% 16.60%Coefficient of Variation 1.09 1.40 **Measures of Downside Risk:**

Shortfall risk 35.00%  
Downside Deviation 10.88%  
Information Ratio 0.27  
Sortino Ratio 0.23  
**Beta** .79

The **average annual returns for the SBMVP and the DJIA are 14.32% and 11.85%** respectively. The **standard deviation of returns for the SBMVP and the DJIA are 15.68% and 16.60%** respectively. Even after allowing 50 basis points per year in transaction costs, the SBMVP average annual returns are 1.97% per year greater than the 30-stock DJIA.

**Table 1** also lists the **shortfall risk, downside deviation, information ratio, Sortino ratio and the Beta of the SBMVP relative to the DJIA for the SBMVP**. These values are 35.00%, 10.88%, 0.27, .23 and .79 respectively.

The **shortfall risk value of 35%** indicates the relative frequency of a fund earning a return below the DJIA rate of return. The **downside deviation value of 10.88%** is measured in units of return. A lower downside deviation indicates a l**ower volatility below the target DJIA returns than the overall volatility of the SBMVP of 15.68%**. The information ratio of .27 indicates a measure of the benchmark relative return gained for taking on benchmark relative risk. A positive ratio means a higher level of differential return over the benchmark. The **Sortino ratio of .23** indicates that there is a positive annual average difference between the SBMVP returns and the DJIA returns per unit of downside deviation. **Empirical beta** in this study is presented as the sensitivity of the SBMVP a portfolio’s returns to the returns of the DJIA. The fact that the e**mpirical beta for the SBMVP strategy is .79** over the study period indicates the SBMVP provides a lower level of systematic risk relative to the DJIA from which it is derived while providing higher overall average annual returns, less risk per unit of return (coefficient of variation) and less downside risk.

**2.** **Table 5** lists the **shortfall risk, downside deviation, information ratio and the Sortino ratio** **relative to a target return of 0 percent (no loss) for the SBMVP and DJIA**.

**Table 5  
Downside Risk Measures for SBMVP and DJIA  
Target Return = 0**

**Measures of Downside Risk SBMVP vs Target = 0**

Shortfall risk 10.00%  
Downside Deviation 5.00%  
Information Ratio 0.91  
Sortino Ratio 2.86

**Measures of Downside Risk DJIA vs Target = 0**

Shortfall risk 20.00%  
Downside Deviation 8.06%  
Information Ratio 0.71  
Sortino Ratio 1.47

For the SBMVP, these values are 10.00%, 5.00%, .91 and 2.86 respectively. The **shortfall risk value of 10%** indicates the percentage of annual returns of the SBMVP that fall below an annual return of 0% (a negative return or loss). The **downside deviation value of 5.00%** is measured in units of return. A lower downside deviation indicates a lower volatility below the target return. The **information ratio of .91** is positive and much the same as a Sharpe ratio since the target return is 0. The **Sortino ratio of 2.86** indicates that there is a positive annual average difference of the fund and the target return of 0 per unit of downside deviation.

For the DJIA, these values are 20.00%, 8.06%, .71 and 1.47 respectively. The **shortfall risk value of 20%** is higher than that for the SBMVP strategy. The **downside deviation value is 8.06%** is higher than the SBMVP and indicates a higher downside volatility below the target return of 0%. The **information ratio of .71** is lower than that for the SBMVP strategy and is essentially a Sharpe ratio. The **Sortino ratio of 2.86 f**or the SBMVP relative to 1.47 for the DJIA indicates that there is a greater positive annual difference between the SBMVP and the target return per unit of downside deviation than that for the DJIA.

**\*All of these downside risk measures support the enhanced performance of the SBMVP relative to the larger DJIA.**

**Disclaimers:**

1. It is acknowledge that weights constraints of other than 20 percent could provide better performance than is reported in this research.

2. While the results of these calculations are shown to enhance portfolio performance in the period analyzed, it is important to realize that they rest on the assumption that historical relationships between individual assets and asset classes will hold in the future. The time period used for collecting investment returns can also impact the results of the analysis. Knowing the limitations of this kind of portfolio analysis is just as important as what it might tell you.

3. It is acknowledge that this study focuses on the SBMVP strategy as applied to the DJIA. The technique of building SBMVPs from a subset of stocks in other indexes or funds may or may not produce enhanced returns relative to the base fund.

**Using Solver in Excel to Find the Minimum Variance Portfolio**

The *Solver* tool in Excel is used to implement mean variance optimization (MVO). To implement MVO in Excel, we must first estimated expected annual returns and the covariance matrix for each set of stocks. In our SBMVP strategy, the average annual historical returns over the previous eight years are used as a basis for the expected future returns for primary set of stocks in each fund. By using these average historical annual returns as a basis for the expected future returns for these stocks, we are making the rather large assumption that these stocks will exhibit the same behavior going forward.

Pairwise covariance estimates are needed for MVO. Covariance is one measure of the degree to which the returns of two risky assets move in tandem. A positive covariance means that asset returns move together, while a negative covariance means returns move inversely (in an opposite direction). Based on the annual return data, we can use Excel’s covariance tool to set up a covariance matrix for multiple securities to see how the annual returns of these stocks have moved in relation to each other over the last seven years. To create the covariance matrix for our primary stocks, we click the Data Analysis button on the Data tab in Excel and choose Covariance from the list. This launches the Covariance dialog box. We use the input range for the annual returns grouped by columns with the stock ticker symbols (labels) in the first row.

When you create a covariance matrix in this manner, half of the resulting table is empty. This is because covariance matrices are symmetric, meaning that the cells in the upper diagonal would be a mirror image of those in the lower diagonal. To fill in the entire covariance matrix, we use the matrix multiplication (MMULT) and transpose (TRANSPOSE) functions in conjunction with one another in Excel. We first select the data range for the covariance matrix, this time *not* including the row with the ticker symbols. After selecting this range, we enter the following formula:

=MMULT(TRANSPOSE(CellXX:CellYY–CellXA:CellYA),(CellXX:CellYY–CellXA:CellYA))/N

Where:

• XX:YY is the data range (excluding the data label row) matrix containing the annual returns for the stocks in our portfolio

• XA:YA is the data range (row) of the average annual returns for the stocks in our portfolio; and

• N is the number of data points (years of annual return data) being used.

Once we have typed in this formula, we enter it using *Shift+Ctrl+Enter;* otherwise, we will get a #VALUE error. When entered correctly, you get a completed covariance matrix.

Using our covariance matrix, we can calculate the SBMVP portfolio standard deviation by multiplying the average annual return for each of the primary stocks by the stock’s respective weighting in the portfolio as determined using the Excel *Solver* function. For example, using the weightings of the individual stocks in the portfolio and the covariance matrix, we can calculate the portfolio standard deviation with this formula:

=SQRT(MMULT(MMULT(TRANSPOSE(CellSA:CellSB),($Cell$EE:$Cell$FF),CellSA:CellSB))

Where:

•SA:SB is the data range (column) for the stock weights as determined by *Solver*.

• EE:FF is the data range (matrix) for the numerical values in the covariance matrix generated by *Excel*.

Once again, it is important to enter the formula for the portfolio standard deviation using *Shift+Ctrl+Enter*. In effect, the formula multiplies the portfolio weights of one pair of stocks and their respective covariance and adds these products together for all the stock pairs and then takes the square root of the total to arrive at the portfolio standard deviation.

**Table 1** describes the necessary parameter values for Solver in determining the stock weights for the ex ante SBMVP. To create the SBMVP for each of the funds using MVO and the list of primary stocks from each fund, we need to minimize the standard deviation cell (the target cell). To do this we select the *Min* button to the right of *Equal To:* in the *Solver* Parameters box so that the *Solver* tool will minimize the function in the target cell (in this case, the standard deviation of the SBMVP). *Solver* derives the minimum variance portfolio in our target cell by adjusting the weights of the primary stocks (CellSA:CellSB) subject to our constraints. The first constraint is that the sum of the weights invested in the SBMVP stocks will equal 1.0 (100 percent). Therefore, we click the *Add* button in the *Solver* Parameters box and add the constraint in the sum of the weights cell box = 1.0. Beyond the constraint that the security weights equal 1.0 (100 percent), we add the constraint requiring that each weighting determined by *Solver* in the range CellSA:CellSB is greater than or equal to zero, to prohibit short selling. In order to implement our small-basket approach, our next constraint caps the maximum weight for an individual stock at 20 percent (suggest a value between 10 and 20 percent to limit the number of resulting stocks in the SBMVP). Click *OK* and click on the *Solve* button. Looking at the range CellSA:CellSB, we see the weights of our primary stocks, as determined by *Solver*, that will give us the lowest possible standard deviation subject to our stated constraints, the SBMVP.

This process is continued in order to find the ex ante weights for the SBMVP for years 1995 through 2014. For example, the ex ante portfolio for 1995 is constructed based on the stock weights from the ex post efficient portfolio derived using the historical return, standard deviation and correlation data from 1987–1994, the ex ante portfolio for 1996 is constructed based on ex post efficient portfolio derived using the data from 1988–1995, and so forth. No short sales are allowed in the SBMVP strategy. The ex ante portfolios further limits the weight in any on single stock to 20 percent of the portfolio. As a result, the ex ante SBMVP generally contains only 5 to 10 stocks.